Schulze method

Zuzanna Opała
Mateusz Sieniawski
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How it works?

Asembler	
Brainfuck	
C++	
D	

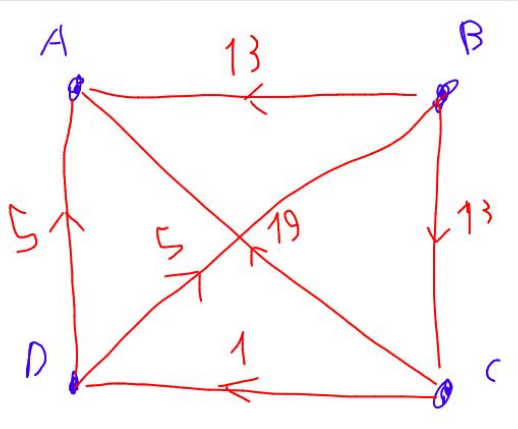
1. Head-to-head comparisons

	7	4	3	5
1st	B	D	C	D
2nd	C	B	D	B
3rd	A	C	A	C
4th	D	A	B	A

2. Digraph

AB	AC	AD	BC	BD	CD
13	19	5	13	5	1

3. Head-to-head strength comparisons



D vs C	D vs B	D vs A
C vs B	**C vs A**	**B vs A**

Formalism

Let **d[V,W]** be the number of voters who prefer candidate V to candidate W

Let **p[A,B]** be a strength of the strongest from all paths between A and B
where strength of the path $(v_0, v_1, \ldots, v_k)$ is $\min(d[v_i, v_{i+1}])$
(if there is no path between A and B then p[A,B] = 0)

Candidate D is better than candidate E if and only if **p[D,E] > p[E,D]**.

Candidate D is a potential winner if and only if $\forall_E$ **p[D,E] ≥ p[E,D]**

Transitivity of p

$$p[X,Y] > p[Y,X] \land p[Y,Z] > p[Z,Y] \Rightarrow p[X,Z] > p[Z,X]$$

Proof

Assume: $p[X,Y] > p[Y,X] \land p[Y,Z] > p[Z,Y] \land p[X,Z] < p[Z,X]$

Lemma: $p[A,C] \geq \min(p[A,B], p[B,C])$

If not, path **A**⇢**B**⇢**C** *would be stronger than the strongest path*

From this we have:

$$\min(p[X,Y], p[Y,Z]) \leq p[X, Z] < p[Z, X]$$
$$\min(p[Y,Z], p[Z,X]) \leq p[Y, X] < p[X, Y]$$
$$\min(p[Z,X], p[X,Y]) \leq p[Z,Y] < p[Y, Z]$$

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If not, path **A**⇢**B**⇢**C** *would be stronger than the strongest path*

From this we have:

$\min(p[X,Y], p[Y,Z]) \leq p[X, Z] < p[Z, X]$ $\quad\quad$ $\min(a, b) < c$

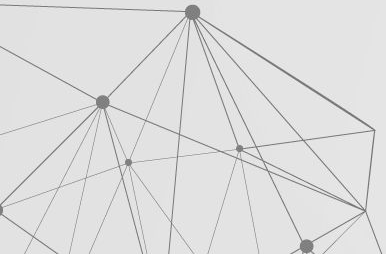
$\min(p[Y,Z], p[Z,X]) \leq p[Y, X] < p[X, Y]$ $\quad\quad$ $\min(b, c) < a$

$\min(p[Z,X], p[X,Y]) \leq p[Z,Y] < p[Y, Z]$ $\quad\quad$ $\min(c, a) < b$

contradiction

Because p (relation of *better*) is transitive
it is guaranteed that there is always at least one candidate D
with p[D,E] >= p[E,D] for every other candidate E

(there is always a potential winner)



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1 # Input: d[i,j], the number of voters who prefer candidate i to candidate j.
2 # Output: p[i,j], the strength of the strongest path from candidate i to candidate j.
3
4 for i from 1 to C
5    for j from 1 to C
6        if (i ≠ j) then
7            if (d[i,j] > d[j,i]) then
8                p[i,j] := d[i,j]
9            else
10               p[i,j] := 0
11
12 for i from 1 to C
13    for j from 1 to C
14        if (i ≠ j) then
15            for k from 1 to C
16                if (i ≠ k and j ≠ k) then
17                    p[j,k] := max (p[j,k], min (p[j,i], p[i,k]))
```

$O(C^3)$

Properties

Condorcet criterion

satisfied

Assume that V is condorcet winner.
Then for all X:

d[V, X] > ½ n, d[X,V] < ½ n, p[V, X] > ½ n, p[X, V] < ½ n

p[V, X] > ½ n > p[X, V]

Condorcet loser criterion

satisfied

Assume that V is condorcet winner.
Then for all X:

d[V, X] < ½ n, d[X,V] > ½ n, p[V, X] < ½ n, p[X, V] > ½ n

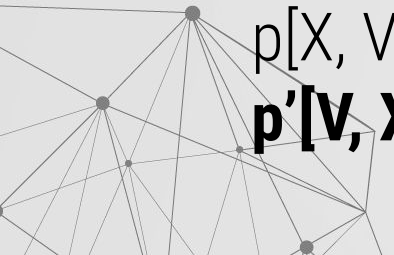
p[V, X] < ½ n < p[X, V]

Monotonic criterion

satisfied

Assume that V is ballot winner - for all X p[V, X] >= p[X, V]
When we move V higher in preferences then:

d[V, X] + 1 >= d’[V, X] >= d[V, X]
d[X, V] - 1 <= d’[X, V] <= d[X, V]
p[V, X] + 1 >= p’[V, X] >= p[V, X]
p[X, V] - 1 <= p’[X, V] <= p[X, V]
p’[V, X] >= p[V, X] >= p[X, V] >= p’[X, V]



- Invulnerability to compromising
- Invulnerability to burying
- Later-no-harm

kewise, since the Schulze method is not a dictatorship and agrees with unanimous votes, Arrow's Theorem implies it fails the criterion

- Independence of irrelevant alternatives

he Schulze method also fails

- Peyton Young's criterion Local Independence of Irrelevant Alternatives.

omparison table [edit]

he following table compares the Schulze method with other preferential single-winner election methods:

Comparison of preferential electoral systems [hide]

System	Mono-tonic	Cond-orcet	Majority	Condorcet loser	Majority loser	Mutual majority	Smith	ISDA	LIIA	Independence of clones	Reversal symmetry	Participation, consistency	Later-no-harm	Later-no-help	Polynomial time
Schulze	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	No	No	No	Yes
Ranked pairs	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	No	No	Yes
Tideman's Alternative	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	No	No	No	No	Yes
Kemeny–Young	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	No	No	No	No
Copeland	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	No	Yes	No	No	No	Yes
Nanson	No	Yes	Yes	Yes	Yes	Yes	Yes	No	No	No	Yes	No	No	No	Yes
Instant-runoff voting	No	No	Yes	Yes	Yes	Yes	No	No	No	Yes	No	No	Yes	Yes	Yes
Borda	Yes	No	No	Yes	Yes	No	No	No	No	No	Yes	Yes	No	Yes	Yes
Baldwin	No	Yes	Yes	Yes	Yes	Yes	Yes	No	No	No	No	No	No	No	Yes
Bucklin	Yes	No	Yes	No	Yes	Yes	No	No	No	No	No	No	No	Yes	Yes
Plurality	Yes	No	Yes	No	No	No	No	No	No	No	No	Yes	Yes	Yes	Yes
Contingent voting	No	No	Yes	Yes	Yes	No	No	No	No	No	No	No	Yes	Yes	Yes
Coombs[4]	No	No	Yes	Yes	Yes	Yes	No	No	No	No	No	No	No	No	Yes
MiniMax	Yes	Yes	Yes	No	No	No	No	No	No	No	No	No	No	No	Yes
Anti-plurality[4]	Yes	No	No	No	Yes	No	No	No	No	No	No	Yes	No	No	Yes
Sri Lankan contingent voting	No	No	Yes	No	No	No	No	No	No	No	No	No	Yes	Yes	Yes
Supplementary voting	No	No	Yes	No	No	No	No	No	No	No	No	No	Yes	Yes	Yes
Dodgson[4]	No	Yes	Yes	No	No	No	No	No	No	No	No	No	No	No	No

he main difference between the Schulze method and the ranked pairs method can be seen in this example:

uppose the MinMax score of a set **X** of candidates is the strength of the strongest pairwise win of a candidate A ∉ **X** against a candidate B ∈ **X**. Then the Schulze method, but not Ranked Pairs, guarantees that the winner is always a candidate of the set with minimum MinMax score.[1]:§4.8 So, in som chulze method minimizes the largest majority that has to be reversed when determining the winner.

n the other hand, Ranked Pairs minimizes the largest majority that has to be reversed to determine the order of finish, in the minlexmax sense. [5] In other words, when Ranked Pairs and the Schulze method produce different orders of finish, for the majorities on which the two orders of finish disagree der reverses a larger majority than the Ranked Pairs order.

istory [edit]

he Schulze method was developed by Markus Schulze in 1997. It was first discussed in public mailing lists in 1997–1998[6] and in 2000.[7] Subsequently, Schulze method users included Debian (2003),[8] Gentoo (2005),[9] Topcoder (2005),[10] Wikimedia (2008),[11] KDE (2008),[12] the Pirate Party of S

Usage

References

- Markus Schulze (2018). The Schulze Method of Voting. *CoRR, abs/1804.02973*. *(https://arxiv.org/abs/1804.02973)*
- Markus Schulze (2010). A new monotonic, clone-independent, reversal symmetric, and condorcet-consistent single-winner election method. *Social Choice and Welfare, 36*(2), 267–303.
- https://en.wikipedia.org/wiki/Schulze_method

THANKS